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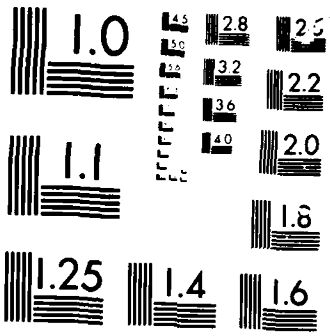

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WEAK CONVERGENCE OF THE VARIATIONS, ITERATED INTEGRALS,

AND DOLÉANS-DADE EXPONENTIALS OF SEQUENCES OF SEMIMARTINGALES

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Abstract

If $X^{(n)}$ is a sequence of semimartingales, converging to a semimartingale X, and such that $[X^{(n)}, X^{(n)}]$ converges to $[X,X]$, then all higher order variations and all the iterated integrals of $X^{(n)}$ converge jointly to the respective functionals of X.

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1. Introduction

A. Let $X_t^{(n)}$ be a sequence of semimartingales, with $t \in [0,1]$, such that

$$X^{(n)} \xrightarrow{w(J_1)} X, \tag{1.1}$$

where X is a semimartingale, and $\xrightarrow{w(J_1)}$ denotes weak convergence on D[0,1] with respect to the J_1-Skorohod topology.

We investigate the convergence of the variations, iterated integrals and Doléans Dade exponentials of $X^{(n)}$, which are defined as follows: For Y a semimartingale,

$$V_k(Y)_t = \begin{cases} Y_t & \text{for } k=1 \\ [Y,Y]_t = \langle Y,Y\rangle_t + \sum_{s\le t} (\Delta Y_s)^2, & \text{for } k=2 \\ \sum_{s\le t} (\Delta Y_s)^k, & \text{for } k\ge 3 \end{cases} \tag{1.2}$$

$$I_k(Y)_t = \begin{cases} Y_t & \text{for } k=1 \\ \int_0^t I_{k-1}(Y)_{s-}\, dY_s, & \text{for } k\ge 2 \end{cases} \tag{1.3}$$

$$E(\lambda Y)_t = \exp[\lambda Y_t - \frac{\lambda^2}{2}[Y,Y]_t] \prod_{s\le t} \ell(\lambda \Delta Y_s), \tag{1.4}$$

where $\ell(x) = (1+x)e^{-x+\frac{x^2}{2}}$.

$V_k(Y)$, $I_k(Y)$ and $E(\lambda Y)$ are called respectively the variations, the iterated integrals and the Doléans-Dade exponential of the semimartingale Y. It is known that V_k, I_k and E are well defined for any semimartingale Y (see Meyer, 1976). These quantities are important in the theory of multiple integration with respect to Y_t.

B. When $X_t^{(n)} = \sum_{i=1}^{[nt]} X_{i,n}$, with $X_{i,n}$ a triangular array, then

$$V_k(X^{(n)})_t = \sum_{i=1}^{[nt]} X_{i,n}^k,$$

$$I_k(\overset{(n)}{X})_t = \sum_{1 \le i_1 < \dots < i_k \le [nt]} X_{i_1,n} \cdots X_{i_k,n},$$

and

$$E(\lambda \overset{(n)}{X})_t = \prod_{i=1}^{[nt]} (1 + \lambda X_{i,n}) = \sum_{k=0}^{[nt]} \lambda^k I_k(\overset{(n)}{X})_t.$$

The problem of the convergence of these "moments", "symmetric statistics", and generating function of the symmetric statistics have been studied in [1],[3-5],[7], and [9].

C. From formula 41.1 of Meyer (1976), it follows that in the semimartingale context, just like in the discrete deterministic case, I_k, $k = 1,\dots, m$ and V_k, $k = 1,\dots, m$ can be represented as polynomials of n variables in one another (the Newton polynomials which relate sums of powers to the sums of products). Thus, the issue of the joint convergence of I_k, $k = 1,\dots, m$, and that of the convergence of V_k, $k = 1,\dots, m$, are equivalent.

D. $\overset{(n)}{X} \xrightarrow{w(J_1)} X$ does not imply in general $[\overset{(n)}{X}, \overset{(n)}{X}] \to [X,X]$, as the following deterministic example from Jacod (1983) shows:

$$\overset{(n)}{X}_t = \sum_{k=1}^{[n^2 t]} \frac{(-1)^k}{n} \text{ converges uniformly to 0, but } [\overset{n}{X},\overset{n}{X}]_t = \sum_{k=1}^{[n^2 t]} \frac{1}{n^2} \to t.$$

E. However, the following result holds:

<u>Theorem 1</u>: The following three statements are equivalent.

$$(1.5) \qquad (\overset{(n)}{X}, [\overset{n}{X},\overset{n}{X}]) \xrightarrow[n\to\infty]{w(J_1)} (X,[X,X]),$$

$$(1.6) \qquad (V_1(\overset{(n)}{X}),\dots, V_m(\overset{(n)}{X})) \xrightarrow[n\to\infty]{w(J_1)} V_1(X),\dots, V_m(X)), \quad \forall m \ge 2,$$

$$(1.7) \qquad (I_1(\overset{(n)}{X}),\dots, I_m(\overset{(n)}{X})) \xrightarrow[n\to\infty]{w(J_1)} I_1(X),\dots, I_m(X)), \quad \forall m \ge 2.$$

They also imply:

$$(1.8) \qquad E(\lambda \overset{(n)}{X}) \xrightarrow{w(J_1)} E(\lambda X), \quad \forall \lambda.$$

Corollary: If

$$X^{(n)} \xrightarrow{w(J_1)} X \tag{1.9}$$

and the condition of Jacod (1983) holds:

$$\lim_{b\to\infty} \sup_{n\to\infty} P\{Var(B^{h,n})_1 > b\} = 0 \tag{1.10}$$

(where h is a truncation function and $(B^{h,n})_t$ is the previsible projection of the truncated semimartingale $\overset{n}{X}$), then (1.5), (1.6), (1.7) and (1.8) hold.

Proof: cf. Jacod (1983), Theorem 5.1.1, (1.9) and (1.10) imply (1.5).

2. Proofs

Introduce the following notation: For any real number x,

$$x^{>a} := x\cdot 1_{\{|x|>a\}}$$

$$x^{\leq a} := x\cdot 1_{\{|x|\leq a\}}$$

We establish now the following:

Lemma 1: a) Suppose $X^{(n)}$ are semimartingales such that

$$\lim_{b\to\infty} \overline{\lim_{n\to\infty}} P\{[X^{(n)}, X^{(n)}]_1 > b\} = 0, \tag{2.1}$$

and let $f(x)$ be any real function such that $f(x) = o(x^2)$, as $x \to 0$. Then, for all ε,

$$\lim_{a\to 0} \overline{\lim_{n\to\infty}} P\{\sum_{s\leq 1} |f(\Delta X_s^{\leq a})| \geq \varepsilon\} = 0. \tag{2.2}$$

b) If the assumptions of a) hold, $X^{(n)} \xrightarrow{w(J_1)} X$ and f is a continuous, vector valued function, then:

$$\sum_{s\leq t} f(\Delta X_s^{(n)}) \xrightarrow{w(J_1)} \sum_{s\leq t} f(\Delta X_s). \tag{2.3}$$

Proof: a) Note first that $\Sigma_{s\le t}|f(\Delta \overset{(n)}{X}_s)| < \infty$, since $\Sigma_{s\le t}\Delta \overset{(n)}{X}_s{}^2 < \infty$. Let now $g(a) = \sup_{|x|\le a}|f(x)|/x^{-2}$. Then,

$$P\{\sum_{s\le 1}|f(\Delta \overset{(n)}{X}_s{}^{\le a})| > \varepsilon\} \le P\{\sum_{s\le 1}(\Delta \overset{(n)}{X}_s{}^{\le a})^2 g(a) > \varepsilon\}$$

$$\le P\{[\overset{(n)}{X}, \overset{(n)}{X}]_1 > \varepsilon/g(a)\}.$$

Since $g(a)\to 0$, (2.2) follows from (2.1).

b) Let $U(X) = \{u > 0 : P\{|\Delta X_t| \ne u, \text{ for all } t\} = 0\}$. $U(X)$ is dense in R_+. For any $a \in U(X)$, and f continuous, the functional

$$S_f^a(Z)_t = \sum_{s\le t} f(\Delta Z_s{}^{>a})$$

is J_1 continuous a.s. (dist (X)). Thus, $\overset{(n)}{X} \xrightarrow{w(J_1)} X$ implies for $a \in U(X)$

$$S_f^a(\overset{(n)}{X}) \xrightarrow{w(J_1)} S_f^a(X).$$

Also,

$$S_f^a(X)_t \xrightarrow[a\to 0]{a.s.\ (J_1)} S_f(X)_t := \sum_{s\le t} f(\Delta X_s).$$

The result follows now by (2.2) and Theorem 4.2 of Billingsley (1968).

Proof of Theorem 1:

By Lemma 1b, we have (1.5) ⇒ (1.6), and in fact the same type of argument yields (1.5) ⇒ (1.8), as follows: Assume for convenience $\lambda = 1$ and $1 \in U(X)$, let

$$f(x) = [\ell n(1+x) - x + \frac{x^2}{2}]\,|_{\{|x|\le 1\}},$$

and let $T : D_{[0,1]} \to D_{[0,1]}$ be defined by:

$$T(Z)_t := \prod_{s\le t} \ell(\Delta Z_s^{>1}) = \prod_{s\le t}(1+\Delta Z_s{}^{>1})\exp\{-\Delta Z_s{}^{>1} + \frac{1}{2}(\Delta Z_s{}^{>1})^2\}.$$

Since the Doléans-Dade exponential

$$E(X)_t = \exp\{X_t - \tfrac{1}{2}[X,X]_t + \sum_{s\le t} f[\Delta X_s^{\le 1}]\} \cdot T(X)_t,$$

it remains only to note that the functional:

$$X^a : D^{(2)}[0,1] \to D^{(4)}[0,1]$$

$$X(Z_1,Z_2) = (Z_1,Z_2,S_f^a(Z_1),T_{Z_1})$$

is continuous a.s., if both spaces are endowed with the respective J_1 topologies. Letting then $a \to 0$, as in the proof of Lemma 1, one gets:

$$\Big(\overset{(n)}{X}_t,\ [\overset{(n)}{X},\overset{(n)}{X}]_t,\ \sum_{s\le t} f(\Delta \overset{(n)}{X}{}_s^{\le 1}),\ \prod_{s\le t} \ell(\Delta \overset{(n)}{X}{}_s^{>1})\Big)$$

$$\xrightarrow{w(J_1)} \Big(X_t,[X,X]_t,\ \sum_{s\le t} f(\Delta X_s^{\le 1}),\ \prod_{s\le t} \ell(\Delta X_s^{>1})\Big),$$

since $\ell n(1+x) - x + \frac{x^2}{2} = o(x^2)$, and since (1.5) implies (2.1). Finally, applying the continuous functional

$$\rho : D^{(4)}_{[0,1]} \to D_{[0,1]},$$

$$\rho(Z_1,Z_2,Z_3,Z_4) = \exp[Z_1 - \tfrac{1}{2}Z_2 + Z_3] \cdot Z_4,$$

we get that

$$E(\lambda \overset{(n)}{X}) \xrightarrow{w(J_1)} E(\lambda X).$$

Since (1.6) is equivalent to (1.7) (by the use of the polynomial mapping), and (1.6) trivially implies (1.5), Theorem 1 is proved. $\square$

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